DSA 595 Bayesian computations for machine learning
Problem set 1

January 15, 2025

1. Derive the posterior distribution for p based on a sample $X_1, \dots, X_n \overset{\text{iid}}{\sim} \text{binomial}(n, p)$, and the prior distribution $p \sim \text{beta}(a, b)$. Assume n, a, b are fixed and known.

2. Derive the posterior distribution for λ based on a sample $X_1, \dots, X_n \overset{\text{iid}}{\sim} \text{Poisson}(\lambda)$, and the prior distribution $\lambda \sim \text{gamma}(\alpha, \beta)$. Assume n, α, β are fixed and known.

3. Derive the posterior distribution for μ based on a sample $X_1, \dots, X_n \overset{\text{iid}}{\sim} \text{normal}(\mu, \sigma^2)$, and the prior distribution $\mu \sim \text{normal}(\mu_0, \sigma_0^2)$. Assume $n, \sigma^2, \mu_0, \sigma_0^2$ are fixed and known.

4. Derive the posterior distribution for σ^2 based on a sample $X_1, \dots, X_n \overset{\text{iid}}{\sim} \text{normal}(\mu, \sigma^2)$, and the prior distribution $\sigma^2 \sim \text{inverse-gamma}(\alpha, \beta)$. Assume $n, \mu, \mu_0, \sigma_0^2$ are fixed and known.

5. Derive the posterior distribution for θ based on a sample $X_1, \dots, X_n \overset{\text{iid}}{\sim} \text{uniform}(0, \theta)$, and the prior distribution $\theta \sim \text{Pareto}(m, \alpha)$. Assume n, m, α are fixed and known.