DSA 595 Bayesian computations for machine learning
Problem set 2

January 22, 2025

1. Derive the posterior distribution for λ based on a sample $X_1, \dots, X_n \overset{\text{iid}}{\sim} \text{exponential}(\lambda)$, and the prior distribution $\lambda \sim \text{Pareto}(m, \alpha)$. Assume n, m, α are fixed and known.

2. Derive the posterior distribution for σ^2 based on a sample $X_1, \dots, X_n \overset{\text{iid}}{\sim} \text{normal}(\mu, \sigma^2)$, and the improper prior density $\pi(\sigma^2) \propto 1/\sigma^2$. Assume n, μ are fixed and known.

3. Derive the posterior distribution for β based on a sample from the linear regression model $Y_i = x_i^\top \beta + U_i$, independently, for $i \in \{1, \dots, n\}$, where $U_1, \dots, U_n \overset{\text{iid}}{\sim} \text{normal}(0, \sigma^2)$, and the prior distribution $\beta \sim \text{normal}_p(0, \tau^2 \cdot I_p)$, where I_p is the $p \times p$ identity matrix. Assume $n, p, \sigma^2, \tau^2, x_1, \dots, x_n$ are fixed and known.

4. Same as the previous problem, except assume the improper prior density $\pi(\beta) \propto 1$.

5. Suppose that $X \sim \text{Poisson}(\theta)$ with prior density $\pi(\theta) = 1/\theta$ for $\theta > 0$. Show that the posterior distribution $\pi(\theta \mid x)$ is not defined for $x = 0$.