DSA 595 Bayesian computations for machine learning
Problem set 8

March 19, 2025

1. Derive the multivariate posterior distribution kernel for (β, σ^2) based on a sample from the linear regression model
$$Y_i = x_i^\top \beta + U_i,$$
for $i \in \{1, \ldots, n\}$, where $U_1, \ldots, U_n \overset{\text{iid}}{\sim} \text{normal}(0, \sigma^2)$, and priors distributions
$$\beta \sim \text{normal}_p(0, \tau^2 \cdot I_p)$$
$$\sigma^2 \sim \text{inverse-gamma}(a, b).$$

2. Write an MH algorithm to draw samples from the posterior $\pi(\beta, \sigma^2 \mid y_1, \ldots, y_n)$. Generate synthetic data from the linear regression model with $p = 4$, and generate the x_i as:
$$x_{i,1} = 1$$
$$x_{i,2} \sim \text{uniform}(0, 1)$$
$$x_{i,3} \sim \text{uniform}(0, 30)$$
$$x_{i,4} \sim \text{uniform}(0, 60),$$
for $i \in \{1, \ldots, n\}$.

3. Modify your Metropolis-Hastings algorithm in problem 2 with the pre-burn-in adaptive proposal scaling and covariance strategy discussed in lecture this week. Demonstrate that you are able to target an acceptance rate close to the range of $(.4, .5)$, post-burn-in.