DSA 595 Bayesian computations for machine learning
Problem set 9

March 26, 2025

1. Derive the conditional posterior distributions for:

    (a) $\mu \mid \sigma^2, y_1, \ldots, y_n$

    (b) $\sigma^2 \mid \mu, y_1, \ldots, y_n$,

    based on a sample $Y_1, \ldots, Y_n \overset{\text{iid}}{\sim} \text{normal}(\mu, \sigma^2)$, and priors distributions

    $$\mu \sim \text{normal}_p(0, \tau^2 \cdot I_p)$$
    $$\sigma^2 \sim \text{inverse-gamma}(\alpha, \beta).$$

2. Write a Gibbs sampler to draw samples from the joint posterior $\pi(\mu, \sigma^2 \mid y_1, \ldots, y_n)$. Generate synthetic data and demonstrate the performance of your algorithm.

3. Present trace plots and histograms of the samples of μ and σ^2.