ST 453 Advanced computing for statistical reasoning
Homework problem set 10

November 11, 2024

No R packages are permitted for use in this assignment.

1. Suppose that $X_1, \ldots, X_{n_x}$ is an iid sample of size n_x with population standard deviation σ_x, and that $Y_1, \ldots, Y_{n_y}$ is an iid sample of size n_y with population standard deviation σ_y. Assume that both samples of data have the same population mean. Construct a permutation test of the hypothesis

$$H_0 : \sigma_x = \sigma_y \quad \text{versus} \quad H_1 : \sigma_x < \sigma_y,$$

and verify that the test gives control over the type 1 error probability for all levels $\alpha \in \{.01, .02, \ldots, .99\}$ by implementing a simulation study. Additionally, provide a histogram of the p-values in some scenario with H_1 true.

2. Assume that $X_i = \mu + U_i$ for $i \in \{1, \ldots, n\}$ with $U_1, \ldots, U_n$ being an iid sample with a continuous density function that is symmetric about 0. The hypothesis

$$H_0 : \mu = 0 \quad \text{versus} \quad H_1 : \mu > 0$$

can be tested non-parametrically with a *sign test*. A sign test is constructed by defining the test statistic $S := \sum_{i=1}^{n} 1\{X_i > 0\}$, and computing a critical value or a p-value according to the distribution of S under the null hypothesis. Determine the distribution of S under the null hypothesis, and then verify that the sign test gives control over the type 1 error probability for all levels $\alpha \in \{.01, .02, \ldots, .99\}$ by implementing a simulation study. Consider the Gaussian and Cauchy distributions for generating $U_1, \ldots, U_n$. In both scenarios, compare the type 1 error probability to that using the t-test.

3. For all levels $\alpha \in \{.01, .02, \ldots, .99\}$ investigate the coverage of a percentile bootstrap confidence interval for the mean of exponentially distributed data with some rate parameter λ.