

ST 453 Advanced computing for statistical reasoning  
Homework problem set 12

November 25, 2024

**No R packages are permitted for use in this assignment.**

1. Let  $Y_1, \dots, Y_n \stackrel{\text{iid}}{\sim} \exp(\lambda)$ . If the value of  $y_n$  is missing, then construct and implement an EM algorithm to find the MLE of  $\lambda$ . Implement a simulation study to repeat this algorithm for a large number of data sets, and plot a histogram of the MLE of  $\lambda$  for each data set.
2. Derive an EM algorithm for estimating the unknown parameters of data from an exponential mixture model

$$X_1, \dots, X_n \stackrel{\text{iid}}{\sim} f(x) = \sum_{j=1}^m p_j \lambda_j e^{-\lambda_j x}$$

by introducing latent variables  $Y_1, \dots, Y_n \in \{1, \dots, m\}$  with  $P(Y_i = j) = p_j$ , for  $j \in \{1, \dots, m\}$ . Implement a simulation study to repeat this algorithm for a large number of data sets, and plot a histogram of the MLE of  $\lambda$  for each data set.

3. Let  $X \sim N(0, 1)$ . Implement a Monte Carlo integration strategy to approximate the mean and variance of the random variable  $Y := X^2$ .
4. Using the probability integral transform, generate data from an  $\exp(\lambda)$  distribution from a sample of observations from the  $\text{uniform}(0, 1)$  distribution.