ST 453 Advanced computing for statistical reasoning
Homework problem set 6

September 30, 2024

No `R` packages are permitted for use in this assignment.

1. Derive the likelihood function for $X_1, \ldots, X_n \overset{\text{iid}}{\sim} \exp(\lambda)$ (i.e., the exponential distribution `https://en.wikipedia.org/wiki/Exponential_distribution`). Plot the likelihood function over a grid of λ values for synthetic data, and indicate with a vertical line where the MLE is. Derive and provide an expression or other mathematical characterization of the MLE.

2. Derive the likelihood function for $X_1, \ldots, X_n \overset{\text{iid}}{\sim} \text{binomial}(n, p)$ (i.e., the binomial distribution `https://en.wikipedia.org/wiki/Binomial_distribution`). Plot the likelihood function over a grid of p values for synthetic data, and indicate with a vertical line where the MLE is. Derive and provide an expression or other mathematical characterization of the MLE.

3. Derive the likelihood function for $X_1, \ldots, X_n \overset{\text{iid}}{\sim} \text{categorical}(k = 3, p_1, p_2, p_3)$ (i.e., the categorical distribution `https://en.wikipedia.org/wiki/Categorical_distribution`). Plot the likelihood function over a grid of p_1 values for synthetic data, and indicate with a vertical line where the MLE is. Derive and provide an expression or other mathematical characterization of the MLE.

4. Derive the likelihood function for $X_1, \ldots, X_n \overset{\text{iid}}{\sim} \text{uniform}(0, \theta)$ (i.e., the continuous uniform distribution `https://en.wikipedia.org/wiki/Continuous_uniform_distribution`). Plot the likelihood function over a grid of θ values for synthetic data, and indicate with a vertical line where the MLE is. Derive and provide an expression or other mathematical characterization of the MLE.

5. Derive the likelihood function for $X_1, \ldots, X_n \overset{\text{iid}}{\sim} \text{Cauchy}(x_0, \gamma)$ (i.e., the Cauchy distribution `https://en.wikipedia.org/wiki/Cauchy_distribution`). Plot the likelihood

function over a grid of x_0 and γ values (separately) for synthetic data, and indicate with a vertical line where the MLE is. Derive and provide an expression or other mathematical characterization of the MLE.