

# ST 495 Advanced computing for statistical methods

## Homework problem set 9

April 9, 2024

**No R packages are permitted for use in this assignment.**

1. Suppose that  $X_1, \dots, X_{n_x}$  is an iid sample of size  $n_x$  with population standard deviation  $\sigma_x$ , and that  $Y_1, \dots, Y_{n_y}$  is an iid sample of size  $n_y$  with population standard deviation  $\sigma_y$ . Assume that both samples of data have the same population mean. Construct a permutation test of the hypothesis

$$H_0 : \sigma_x = \sigma_y \quad \text{versus} \quad H_1 : \sigma_x < \sigma_y,$$

and verify that the test gives control over the type 1 error probability for all levels  $\alpha \in \{.01, .02, \dots, .99\}$  by implementing a simulation study. Additionally, provide a histogram of the p-values in some scenario with  $H_1$  true.

2. Assume that  $X_i = \mu + U_i$  for  $i \in \{1, \dots, n\}$  with  $U_1, \dots, U_n$  being an iid sample with a continuous density function that is symmetric about 0. The hypothesis

$$H_0 : \mu = 0 \quad \text{versus} \quad H_1 : \mu > 0$$

can be tested non-parametrically with a *sign test*. A sign test is constructed by defining the test statistic  $S := \sum_{i=1}^n 1\{X_i > 0\}$ , and computing a critical value or a p-value according to the distribution of  $S$  under the null hypothesis. Determine the distribution of  $S$  under the null hypothesis, and then verify that the sign test gives control over the type 1 error probability for all levels  $\alpha \in \{.01, .02, \dots, .99\}$  by implementing a simulation study. Consider the Gaussian and Cauchy distributions for generating  $U_1, \dots, U_n$ . In both scenarios, compare the type 1 error probability to that using the t-test.

3. For all levels  $\alpha \in \{.01, .02, \dots, .99\}$  investigate the coverage of a percentile bootstrap confidence interval for the mean of exponentially distributed data with some rate parameter  $\lambda$ .