ST 502; Homework 9

October 30, 2019

1. In whatever language you prefer, write a Gibbs sampler to estimate the posterior distribution of μ and σ^2 using the conditional posteriors derived in class. Recall from class that we assumed $X_1, \ldots, X_n \overset{\text{iid}}{\sim} \text{normal}(\mu, \sigma^2)$, and considered the priors $\mu \sim \text{normal}(0, \tau^2)$ and $\sigma^2 \sim \text{inverse-gamma}(\alpha, \beta)$. You will need to specify true values of μ_0 and σ_0^2, and to generate a data set $x_1, \ldots, x_n$. For this exercise, knowledge of the true parameter values will allow you to check if your Gibbs sampler is working or not.

2. Suppose that $Y_1, \ldots, Y_n \overset{\text{iid}}{\sim} \text{uniform}(0, \theta)$.

   (a) Given an expression for the likelihood function of this data.

   (b) What is the conjugate prior for this likelihood function? What is the resulting posterior distribution?

   (c) Pretend that you do not know the normalizing constants for this posterior distribution. In whatever language you prefer, write a Metropolis Hastings algorithm to estimate the posterior distribution of θ. Use a Gaussian random walk proposal distribution (i.e., $\theta^{(t+1)} = \theta^{(t)} + \text{normal}(0, \tau^2)$). Experiment with different values of τ^2 to find a value that gives you an acceptance proportion between .3 and .5.

   Plot your resulting trace plot and histogram, and overlay your trace plot with the true posterior density function you derived in the previous part.

To submit your work, plot and print out your trace plots and histograms, along with the printed code from your script file. Please do not email your homework.