ST 705 Linear models and variance components
Homework problem set 5

February 9, 2024

1. Monahan exercise 2.8.
2. Monahan exercise 2.9.
3. In lecture, we proved a lemma that $(X'X)^g X'$ is a generalized inverse of X.
   (a) Verify that $X(X'X)^g$ is a generalized inverse of X'.
   (b) We proved that $P_X := X(X'X)^g X'$ is the unique symmetric projection onto $\text{col}(X)$. Is $(X'X)^g X'$ the unique generalized inverse of X?
4. Show that $I_n - P_X$ is the unique symmetric projection matrix onto $\text{null}(X')$.
5. Let $X = QR$ where Q has orthonormal columns. Prove that if $\text{rank}(X) = \text{rank}(Q)$, then $P_X = QQ'$.
6. Let $Q = X(X'V^{-1}X)^g X'V^{-1}$, with $V > 0$ and symmetric, and show that Q is a projection onto $\text{col}(X)$.
7. Prove that if a (symmetric) matrix is positive definite, then all of its eigenvalues are greater than zero.