ST 705 Linear models and variance components
Homework problem set 8

March 19, 2024

1. Monahan exercise 3.6.

2. Monahan exercise 3.7.

3. Prove that if $\lambda^{(1)\prime}\beta, \ldots, \lambda^{(k)\prime}\beta$ are estimable, then so is

$$\sum_{j=1}^{k} d_j \lambda^{(j)\prime}\beta,$$

   for any scalar constants $d_1, \ldots, d_k$.

4. Assume that $Y = X\beta + U$, where X is an $n \times p$ matrix with $\mathrm{rank}(X) = k < p$, and assume $\lambda'\beta$ is estimable.

   (a) Construct an argument to determine the rank of the matrix $\begin{pmatrix} X \\ \lambda' \end{pmatrix}$.

   (b) Construct an argument to determine the rank of the matrix $\begin{pmatrix} X \\ \lambda'(I - P_{X'}) \end{pmatrix}$.

5. Let X be an $n \times p$ matrix with $\mathrm{rank}(X) = r$, and C be a $(p - r) \times p$ matrix with $\mathrm{rank}(C) = p - r$, such that $\mathrm{col}(X') \cap \mathrm{col}(C') = \{0\}$. Show that

$$\mathrm{rank}\begin{pmatrix} X \\ C \end{pmatrix} = p.$$

6. Let X be an $n \times p$ matrix with $\mathrm{rank}(X) = r$, and C be a $(p - r) \times p$ matrix with $\mathrm{rank}(C) = p - r$, such that $\mathrm{col}(X') \cap \mathrm{col}(C') = \{0\}$. Show that $C(X'X + C'C)^{-1}C' = I_{p-r}$.