

# ST 705 Linear models and variance components

## Lab practice problem set 11

April 9, 2024

1. Recall the definition of the multivariate normal distribution from class:

**Definition 1** *The  $p$ -dimensional random vector  $Y$  is said to follow the multivariate normal distribution with mean  $\mu$  and covariance matrix  $\Sigma$  if for every  $p$ -dimensional vector  $v$  such that  $v'\Sigma v \neq 0$ ,*

$$v'Y \sim N(v'\mu, v'\Sigma v).$$

Denote  $Y \sim N_p(\mu, \Sigma)$ . ■

Prove that if  $\Sigma$  is nonsingular, then  $Y \sim N_p(\mu, \Sigma)$  if and only if  $Y$  has density,

$$f(y) = \det(2\pi\Sigma)^{-\frac{1}{2}} e^{-\frac{1}{2}(y-\mu)'\Sigma^{-1}(y-\mu)}.$$