ST 705 Linear models and variance components
Lab practice problem set 2

January 16, 2024

1. Show that the covariance function defined for $X, Y \in \mathbb{R}^p$ by

$$\mathrm{Cov}(X, Y) := E[(X - E(X))(Y - E(Y))']$$

satisfies the following properties. For random variables $X, Y, Z \in \mathbb{R}^p$ with finite covariance, and any $c \in \mathbb{R}$,

(a) $\mathrm{Cov}(X + Y, Z) = \mathrm{Cov}(X, Z) + \mathrm{Cov}(Y, Z)$

(b) $\mathrm{Cov}(cX, Y) = c \cdot \mathrm{Cov}(X, Y)$

(c) $\mathrm{Cov}(X, Y)^* = \mathrm{Cov}(Y, X)$

(d) $\mathrm{Cov}(X, X) \geq 0$ for all X, and $\mathrm{Cov}(X, X) = 0$ implies that X is constant a.s.

Then, deduce that if $p = 1$ the covariance is an inner product over some (quotient) vector space, and if $p > 1$ the the function $f(X, Y) := \mathrm{tr}\big(\mathrm{Cov}(X, Y)\big)$ is an inner product.