

ST 705 Linear models and variance components

Lab practice problem set 3

January 24, 2024

1. Let $X \in \mathbb{R}^{n \times p}$, $u \in \mathbb{R}^n$, and $v \in \mathbb{R}^p$.

(a) Prove that

$$|u'Xv| \leq \left(\max_{1 \leq j \leq p} \left\{ \sum_{i=1}^n |X_{i,j}| \right\} \right)^{\frac{1}{2}} \left(\max_{1 \leq i \leq n} \left\{ \sum_{j=1}^p |X_{i,j}| \right\} \right)^{\frac{1}{2}} \cdot \|u\|_2 \cdot \|v\|_2.$$

(b) Show that the Cauchy-Schwarz inequality is a special case of the inequality given in part (a).