# ST 705 Linear models and variance components Lab practice problem set 9 

March 26, 2024

1. Show that if $X$ is a $p$-dimensional random vector with mean $\mu$ and covariance $\Sigma, A$ is a $p \times p$ matrix, and $Y=X^{\prime} A X$, then $E(Y)=\operatorname{tr}(A \Sigma)+\mu^{\prime} A \mu$.
2. For a random vector $Y$, with finite second moment, verify the following properties.
(a) $\mathrm{E}\left(a^{\prime} Y\right)=a^{\prime} \mathrm{E}(Y)$, for a fixed vector $a$.
(b) $\operatorname{Var}\left(a^{\prime} Y\right)=a^{\prime} \operatorname{Var}(Y) a$, for a fixed vector $a$.
(c) $\operatorname{Cov}\left(a^{\prime} Y, c^{\prime} Y\right)=a^{\prime} \operatorname{Var}(Y) c$, for fixed vectors $a$ and $c$.
(d) $\operatorname{Var}\left(A^{\prime} Y\right)=A^{\prime} \operatorname{Var}(Y) A$, for a fixed matrix $A$.
